THE REFRACTIVE INDEX AND THE ABSORPTION INDEX FOR THE PROPAGATION OF RADIO WAVE IN THE IONOSPHERE IN SOME SPECIAL CASES

Y. S. N. MURTY

DEFENCE ELECTRONICS RESEARCH LABORATORY, HYDERABAD

and

S. R. KHASTGIR

BOSE INSTITUTE, CALCUTTA.

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ABSTRACT. The analytical expressions for the refractive index and the absorption index of the ionosphere, obtained by Murty and Khastgir (1963) have been considered for the special cases, when (*i*) $H=0$, (*ii*) $\nu=0$, (*iii*) $N=0$ and (*iv*) $p_0^2=p^2$. The longitudinal case, the transverse case, the *quasi*-transverse case and the case when $\nu<<p'$ have also been discussed. In the case when $H=0$, the expressions are reduced to those obtained by Mitra (1952) following Appleton and Chapman (1932). In the case when $\nu=0$, the expressions are the same as those corresponding to the limiting case discussed by Ratcliffe (1933). In the *quasi*-transverse case and in the case, when $\nu<<p'$, it has been shown that the analytical expressions are reduced to those given earlier by Whitehead (1952, 1956).

It has been shown that the effect of the earth's magnetic field is not only to cause bi-refringence but also to change the values of the refractive index and the absorption index for both the ordinary and the extraordinary modes. Taking collisions into account, Ratcliffe's conclusion that the refractive index in the absence of the earth's magnetic field is minimum, when $p_0^2/p^2=2$ and that it is zero when $p_0^2/p^2=1$ for $\nu=0$ has been confirmed.

INTRODUCTION

Murty and Khastgir (1963) deduced general analytical expressions for the refractive index and the absorption index of the ionosphere from the Appleton-Hartree magneto-ionic formulae. In the present paper, some special cases have been considered. Some of these formulae obtained from the general analytical expressions under specified conditions have been found to agree with the formulae given by previous workers. The applications of the analytical expressions for the refractive index and the absorption index have also been discussed.

EXPRESSIONS FOR REFRACTIVE INDEX AND ABSORPTION INDEX IN SOME SPECIAL CASES

The general expressions for the refractive index μ and the absorption index χ, as given by Murty and Khastgir (1963) are :

$$\mu^2 = \frac{1}{2}\left[\sqrt{1+\frac{1+2a}{a^2+b^2}} + \left(1+\frac{a}{a^2+b^2}\right)\right] \quad \dots \ (1)$$

and

$$\chi^2 = \frac{1}{2}\left[\sqrt{1+\frac{1+2a}{a^2+b^2}} - \left(1+\frac{a}{a^2+b^2}\right)\right] \quad \dots \ (2)$$

where

$$a = \alpha - s\gamma_L \quad \dots \ (3)$$

$$b = \beta + r\gamma_L \quad \dots \ (4)$$

$$r = \rho \sin\phi \quad \dots \ (5)$$

$$s = \rho \cos\phi \quad \dots \ (6)$$

ρ = ratio of the amplitudes of the normal and the abnormal components of the magnetic vector of the radio-waves*

ϕ = phase-difference between the two components

$$\cos 2\phi = a' - \sqrt{(1+a')^2 - 4a'b'} \quad (7)$$

$$a' = \frac{\nu_c^2}{\nu^2+p'^2} \quad \dots \ (8)$$

$$b' = \frac{\nu^2}{\nu^2+p'^2} \quad \dots \ (9)$$

ν = electron collisional frequency

ν_c = critical collisional frequency $= p_H \cdot \dfrac{\sin^2\theta}{2\cos\theta}$

$$p' = p\left(1 - \frac{p_0^2}{p^2}\right), \quad p_H = \frac{eH}{mc}, \quad p_0^2 = \frac{4\pi Ne^2}{m}$$

p = angular frequency of the radio-wave

H = earth's magnetic field

N = electron number density

*It has been shown by Murty & Khastgir (1959) that

$$\rho = \frac{a'}{\nu_c}\left[\frac{\nu}{\cos\phi} - \frac{P'}{\sin\phi}\right] \text{ and } \rho^{-1} = \frac{a'}{\nu_c}\left[\frac{\nu}{\cos\phi} + \frac{p'}{\sin\phi}\right]$$

θ angle between the positive direction of the magnetic field and the direction of wave propagation

e, m = charge and mass of an electron

and c = velocity of light.

When $\frac{1+2a}{a^2+b^2} << 1$, the expressions (1) and (2) are reduced to

$$\mu^2 = 1+\frac{1+4a}{4(a^2+b^2)} \quad \ldots \quad (1a)$$

and

$$\chi^2 = \frac{1}{4(a^2+b^2)} \quad (2a)$$

When a is negative, (1a) can be written as

$$\mu^2 = 1-\frac{4|a|-1}{4(a^2+b^2)} \quad (1b)$$

We shall now consider certain special cases :

Case I—When the earth's magnetic field H is zero

In this case, we find from (3) and (4) that $a = \alpha$ and $b = \beta$, so that we can write

$$\mu^2_{H=0} = \frac{1}{2}\left[\sqrt{1+\frac{1+2\alpha}{\alpha^2+\beta^2}}+\left(1+\frac{\alpha}{\alpha^2+\beta^2}\right)\right] \quad (10)$$

and

$$\chi^2_{H=0} = \frac{1}{2}\left[\sqrt{1+\frac{1+2\alpha}{\alpha^2+\beta^2}}-\left(1+\frac{\alpha}{\alpha^2+\beta^2}\right)\right] \quad \ldots \quad (11)$$

These expressions for $\mu^2_{H=0}$ and $\chi^2_{H=0}$ are the same as those obtained by Mitra (1952) from the expressions for the electrical conductivity of an absorbing medium given by Appleton and Chapman (1932).

When, however, $\frac{1+2\alpha}{\alpha^2+\beta^2} << 1$, we have

$$\mu^2_{H=0} = 1+\frac{1+4\alpha}{4(\alpha^2+\beta^2)} \quad \ldots \quad (10a)$$

and

$$\chi^2_{H=0} = \frac{1}{4(\alpha^2+\beta^2)} \quad \ldots \quad (11a)$$

Since α is negative, (10*a*) can be written as :

$$\mu^2_{H=0} = 1 - \frac{4|\alpha|-1}{4(\alpha^2+\beta^2)} \qquad \dots \text{(10b)}$$

Case II—When the electron collisional frequency ν is zero

In this case, we have $\beta = 0$ and $b' = 0$, so that from (7) $\phi = \frac{\pi}{2}$ and hence $r = 0$ and $b = 0$. We can therefore write equations (1) and (2) as :

$$\mu^2_{\nu=0} = \left(1+\frac{1}{a}\right) = 1 + \frac{1}{\alpha - s\gamma} \qquad \dots \text{(12)}$$

and

$$\chi^2_{\nu=0} = 0 \qquad \dots \text{(13)}$$

The wave-polarization has been taken as $R = r+is = \rho e^{i\varphi}$, where r and s are defined by (5) and (6), ρ is the ratio of the amplitudes of the normal and the abnormal components of the magnetic vector of the wave and ϕ the phase-difference between them. Since for $\nu = 0$, $r = 0$, the Appleton-Hartree formula for the wave-polarization can be written as :

$$R = is = \frac{1}{i\gamma_L}\left[-\frac{\gamma_T^2}{2(1+\alpha+i\beta)} \pm \sqrt{\frac{\gamma_T^4}{4(1+\alpha+i\beta)^2}+\gamma_L^2}\right] \qquad \dots \text{(14)}$$

In (14) the negative sign of the electronic charge is taken into account.

Thus from (12) and (14), we get

$$\mu^2_{\nu=0} = 1 + \frac{1}{\alpha - \frac{\gamma_T^2}{2(1+\alpha)} \pm \sqrt{\frac{\gamma_T^4}{4(1+\alpha)^2}+\gamma_L^2}} \qquad \dots \text{(12a)}$$

This was discussed by Ratcliffe (1933).

From equations (1) and (12), it is found that the value of the refractive index in the absence of collisions is greater than the value in the presence of collisions.

Case III—When the electron number density N is zero

In this case $\alpha \to \infty$ and $\beta \to \infty$

and hence $\qquad a \to \infty$ and $b \to \infty$

Thus we get

$$\mu^2_{N=0} = 1 \qquad \dots \text{(15)}$$

$$\chi^2_{N=0} = 0 \qquad \dots \text{(16)}$$

Case IV—When $p^2_0 = p^2$

In this case, we have $p' = 0$ and $b' = 1$, so that from (7) $\phi = 0$ and hence

$s = 0, \gamma = \rho$ and $a = \alpha$. Further $\alpha = -1$ and $\beta = \frac{\nu}{p}$. Hence the equations (1) and (2) are reduced to

$$\mu^2_{p_0{}^2=p^2} = \frac{1}{2}\left[\sqrt{1-\frac{1}{1+b^2}} + \left(1-\frac{1}{1+b^2}\right)\right] \quad \dots \quad (17)$$

and
$$\chi^2_{p_0{}^2=p^2} = \frac{1}{2}\left[\sqrt{1-\frac{1}{1+b^2}} - \left(1-\frac{1}{1+b^2}\right)\right] \quad \dots \quad (18)$$

It is seen from (17) that the value of $\mu^2_{p_0{}^2=p^2}$ does not become zero at the poin $p^2{}_0 = p^2$, when the collisional effects are predominant.

Case V—Longitudinal Case (i.e. when $\theta = 0$, $\gamma_T = 0$ and $\gamma_L = \gamma$)

In this case, we have $\nu_c = 0$, $a' = 0$, $\phi = \frac{\pi}{2}$. Hence $r = 0$, $b = \beta$ and $R = is$. Further with the help of (14), we get $s = \pm 1$, so that $a = \alpha \pm \gamma$. The equations (1) and (2) can therefore be written as :

$$\mu^2_{\theta=0} = \frac{1}{2}\left[\sqrt{1+\frac{2a}{a^2+\beta^2}} + \left(1+\frac{a}{a^2+\beta^2}\right)\right] \quad \dots \quad (19)$$

and
$$\chi^2_{\theta=0} = \frac{1}{2}\left[\sqrt{1+\frac{2a}{a^2+\beta^2}} - \left(1+\frac{a}{a^2+\beta^2}\right)\right] \quad \dots \quad (20)$$

In this case the values of μ and χ are predominantly affected by the earth's magnetic field.

Case VI—Transverse Case (i.e., when $\theta = 90°$, $\gamma_L = 0$ and $\gamma_T = \gamma$)

When $\theta = 90°$, we have :

(a) $s\gamma_{.} = 0, \quad r\gamma_L = 0$

(b) $s\gamma_L = \frac{\gamma^2(1+\alpha)}{(1+\alpha)^2+\beta^2}, \quad r\gamma_L = \frac{\gamma^2\beta}{(1+\alpha)^2+\beta^2}$

Therefore we get

(*a*) $a = \alpha, \quad b = \beta$

(b) $a = \alpha - \frac{\gamma^2(1+\alpha),}{(1+\alpha)^2+\beta^2} \quad b = \beta\left[1+\frac{\gamma^2}{(1+\alpha)^2+\beta^2}\right]$

Using the values of $a_{\theta=90°}$ and $b_{\theta=90°}$ in equations (1) and (2) we get the expressions for $\mu^2{}_{\theta=90°}$ and $\chi^2{}_{\theta=90°}$. In the transverse case, when $\theta = 90°$, we find that one set of the values of $\mu^2{}_{\theta=90°}$ and $\chi^2{}_{\theta=90°}$, are the same as $\mu^2{}_{H=0}$ and $\chi^2{}_{H=0}$ discussed earlier in Case I.

Case VII—Quasi-transverse Case

In the *quasi*-transverse case, $\frac{\gamma_T^4}{4(1+\alpha+i\beta)^2} >> \gamma_L^2$, so that from (14), we get

$$i\gamma_L R_{QT} = -\frac{\gamma_T^2}{2(1+\alpha+i\beta)} \pm \sqrt{\frac{\gamma_T^4}{4(1+\alpha+iB)^2} + \gamma_L^2}$$

$$= -\frac{\gamma_T^2}{2(1+\alpha+i\beta)} \pm \left|\frac{\gamma_T^2}{2(1+\alpha+i\beta)}\right| \sqrt{1+\frac{4\gamma_L^2(1+\alpha+i\beta)^2}{\gamma_T^4}}$$

$$= -\frac{\gamma_T^2}{2(1+\alpha+i\beta)} \pm \left|\frac{\gamma_T^2}{2(1+\alpha+i\beta)}\right| \left[1+\frac{2\gamma_L^2(1+\alpha+i\beta)^2}{\gamma_T^4}\right] \quad \dots(21)$$

Now $\frac{1}{1+\alpha+i\beta} = \frac{(1+\alpha)-i\beta}{(1+\alpha)^2+\beta^2}$. Below the reflection level, $p_0^2 = p^2$, $(1+\alpha)$ is negative. Hence $\left(\frac{1}{1+\alpha+i\beta}\right)$ is also negative in that region. Thus for the ordinary mode, below the level, $p_0^2 = p^2$, we have from (21) :

$$i\gamma_L R_{OT} = i\gamma_L(r_{OT}+s_{QT})$$

$$= -\frac{\gamma_T^2}{2(1+\alpha+i\beta)} - \left|\frac{\gamma_T^2}{2(1+\alpha+i\beta)}\right| \left[1+\frac{2\gamma_L^2(1+\alpha+i\beta)^2}{\gamma_T^4}\right]$$

$$= \left|\frac{\gamma_T^2}{2(1+\alpha+i\beta)}\right| - \left|\frac{\gamma_T^2}{2(1+\alpha+i\beta)}\right| + \frac{\gamma_L^2}{\gamma_T^2}\left|(1+\alpha+i\beta)\right|$$

$$= \frac{\gamma_L^2}{\gamma_T^2}\left|(1+\alpha+i\beta)\right| \quad \dots (22)$$

Hence from (22), we get

$$\gamma_L r_{OT} = \beta \cot^2\theta \text{ and } \gamma_L s_Q = -(1+\alpha)\cot^2\theta \quad \dots (23)$$

Therefore we get

$$a_{QT} = \alpha+(1+\alpha)\cot^2\theta \quad \text{and} \quad b_{QT} = \beta \operatorname{cosec}^2\theta \quad \dots (24)$$

The Appleton-Hartree formula for the square of the complex refractive index can be written as :

$$m^2 = (\mu-i\chi)^2 = 1+\frac{1}{(\alpha+i\beta)+i\gamma_L(r+is)} = 1+\frac{a-ib}{a^2+b^2} \quad \dots (25)$$

Hence we get $$\mu^2-\chi^2 = 1+\frac{a}{a^2+b^2} \qquad \ldots \text{(26a)}$$

and $$2\mu\chi = \frac{}{a^2+b^2} \qquad \ldots \text{(26b)}$$

Thus when $\chi^2 << \mu^2$ and $\frac{\nu}{p} << 1$, we can write from equations (24) and (26a)

$$\mu_{OT}^2 \approx 1+\frac{1}{\alpha+(1+\alpha)\cot^2\theta} \text{ (since } \beta=0) \qquad \ldots \text{(27)}$$

$$\approx (1-x)\operatorname{cosec}^2\theta$$

where $$x = -\frac{1}{}.$$

Similarly, the expression for χ_{OT} can be written from (24) and (26b)

$$\chi_{OT} = \frac{1}{2\mu_{OT}} \cdot \frac{\beta \operatorname{cosec}^2\theta}{\{1+(1+\alpha)\cot^2\theta\}^2+\beta^2\operatorname{cosec}^4\theta}$$

$$\frac{x^2\beta\operatorname{cosec}\theta}{2\sqrt{(1-x)}} \qquad \ldots \text{(28)}$$

The expressions (27) and (28) are the same as those given earlier by Whitehead (1952).

Case VIII—When $\nu << p'$

In this case, we can write equation (14) for the ordinary wave as :

$$iR \approx \frac{\nu_c(p'+i\nu)}{p'^2} - \sqrt{\frac{\nu_c^2(p'+i\nu)^2}{p'^4}+1}$$

$$\approx \frac{\nu_c(p'+i\nu)}{p'^2} - \sqrt{(1+a')+\frac{2i\nu a'}{p'}}$$

i.e., $$i(r+is) \approx \frac{a'(p'+i\nu)}{\nu_c} - \sqrt{1+a'}\left\{1+\frac{i\nu a'}{p'(1+a')}\right\}$$

i.e., $$r_{\nu<<p'} \approx \frac{\nu\sqrt{a'}}{p'} - \frac{a'\nu}{p'\sqrt{1+a'}} \qquad \ldots \text{(29)}$$

and $$s_{\nu<<p'} \approx \sqrt{1+a'} - \sqrt{a'} \qquad \ldots \text{(30)}$$

Therefore we get,

$$\alpha_{\nu<<p'} = \alpha-\{\sqrt{1+a'}-\sqrt{a'}\}\gamma_L \qquad \dots (31)$$

and
$$b_{\nu<<p'} = \beta+\frac{\nu\sqrt{a'}}{p'}\left\{1-\frac{\sqrt{a'}}{\sqrt{1+a'}}\right\}\gamma_L \qquad \dots (32)$$

Hence the expression for $\mu^2_{\nu<<p'}$ can be written from equation (1) as :

$$\mu^2_{\nu<<p'} \approx 1+\frac{1}{a} \approx 1+\frac{1}{\alpha-\{\sqrt{1+a'}-\sqrt{a'}\}\gamma_L} \qquad \dots (33)$$

and from equation (2) we have :

$$\chi_{\nu<<p'} \approx \frac{1}{2\mu_{\nu<<p'}}\cdot\frac{b_{\nu<<p'}}{a^2_{\nu<<p'}}$$

$$\approx \frac{\beta+\frac{\nu\sqrt{a'}}{p'}\left\{1-\frac{\sqrt{a'}}{\sqrt{1+a'}}\right\}\gamma_L}{2[\alpha-\{\sqrt{1+a'}-\sqrt{a'}\}\gamma_L]^2\sqrt{1+\frac{1}{\alpha-\{\sqrt{1+a'}-\sqrt{a'}\}\gamma_L}}} \qquad \dots (34)$$

The expressions for $\mu^2_{\nu<<p'}$, and $\chi_{\nu<<p'}$ are the same as those obtained earlier by Whitehead (1956) (*vide* Appendix).

APPLICATIONS OF THE EXPRESSIONS FOR μ AND χ

(i) *Computation of the dispersion and absorption curves* :

The dispersion and absorption curves showing the variation of μ and χ as function of the electron number density, for some assumed values of collisional frequency and earth's magnetic field conditions can be easily computed for different wave frequencies for both ordinary and extraordinary modes of propagation by using the expressions for μ and χ given in equations (1) and (2). The values of ρ and ϕ are first computed by using the expressions given earlier by Murty and Khastgir (1959, 1960a). Then the values of r and s, corresponding to either ordinary or extraordinary mode are evaluated using equations (5) and (6). From equation (3) and (4) we can then get the values of a_0, b_0 or a_x, b_x which can be used in equations (1) and (2) to get the value of μ_0, χ_0 or μ_x, χ_x.

(ii) *Evaluation of the group-refractive index of the ionospheric medium*

The group refractive index μ' can be written as,

$$\mu' = \mu+p\frac{\partial\mu}{\partial p} \qquad \dots (35)$$

Equation (35) can also be written as :

$$\mu\mu' = \frac{1}{2p}(p^2\mu^2) \qquad \dots (36)$$

Using the general expression for μ^2 given in equation (1), the expression for $\frac{d}{dp}$ $(p^2\mu^2)$ can be obtained. Hence by using equation (36), the general expression for the group refractive index μ' can be deduced. The general expression for μ' has already been obtained by Murty and Khastgir (1960c, 1961).

(iii) *Effect of earth's magnetic field on the refractive index and the absorption index of the ionosphere* :

As the negative sign of the electronic charge has already been taken into account, γ_L should be taken as positive. Considering now the negative sign of α, we get from (3), (4), (5) and (6)

$$a^2+b^2 = \alpha^2+\beta^2+\rho\gamma_L[2(\alpha \sin\phi+\beta\cos\phi)+\rho\gamma_L] \quad \dots \quad (37)$$

We shall now discuss the cases of the ordinary and the extraordinary modes of propagation separately.

Ordinary Mode

Let us consider the ordinary mode in the northern hemisphere below the ionospheric reflection level, $p^2_0 = p^2$. We then have $\rho < 1$ and ϕ is in the first quadrant $\left[0 < \phi < \frac{\pi}{2}\right]^*$, so that $\sin\phi$ and $\cos\phi$ (also s and r) are positive. Accordingly for the ordinary mode, under the conditions, we get from (37)

$$a^2_0+b_0^2 > \alpha^2+\beta^2$$

We have put : $a = \alpha - s\gamma_L$. Here α is negative and s positive so that $a = -|\alpha|-|s\gamma_L|$. Hence a is negative and the approximate value of the refractive index would be given by (1b). Taking a 100 m-wave at the reflection leave $p_0^2 = p^2$, the scalar magnitude of α is unity and is very much greater than $\rho\gamma_L$ or $s\gamma_L$. Comparing now (1b) with (10b), it is found that for the ordinary mode under the specified conditions, the effect of the earth's magnetic field would be to increase the refractive index. From (2a) and (11a) it is evident that the effect of the earth's magnetic field would be to decrease the absorption index for the ordinary mode.

Extraordinary mode

In the northern hemisphere for the extraordinary mode, below the ionospheric reflection level, $\rho > 1$ and ϕ is in the fourth quadrant $\left(\frac{3\pi}{2} < \phi < 2\pi\right)^*$ so that $\sin\phi$ and s are negative and $\cos\phi$ and r positive. Accordingly for the extraordinary mode under the conditions we get from (37) :

$$a_X^2+b_X^2 = \alpha^2+\beta^2-\rho\gamma_L[2(\alpha\sin\phi-\beta\cos\phi)-\rho\gamma_L] \quad \dots \quad (38)$$

Thus $a_X^2+b_X^2 < \alpha^2+\beta^2$, provided $\rho\gamma_L < 2(\alpha\sin\phi-\beta\cos\phi)$. We have put $a = \alpha - s\gamma_L$. Here α and s are negative, so that $a = |s\gamma_L|-|\alpha|$. When $|s\gamma_L| < |\alpha|$, a is negative and the refractive index would then be given by (1b). Taking a

* *Vide* Table I of the paper by Murty & Kastgir (1960b).

100 m-wave at the reflection level, $p_0^2 = p^2$, as before, and comparing (1b) with (10b), it is found that for the extraordinary mode under the conditions, the effect of the earth's magnetic field would be to decrease the refractive index, since $a_X^2 + b_X^2 < \alpha^2 + \beta^2$ for $p\gamma_L < 2(\alpha \sin\phi - \beta\cos\phi)$. From (2a) and (11a) it is evident that the effect of the earth's magnetic field would be to increase the absorption index for the extraordinary mode.

(iv) *Conditions of reflection from the ionosphere in the absence of the earth's magnetic field* :

Since $\beta = \frac{p\nu}{p_0^2}$ $\alpha\nu$, the expression for $\mu^2_{H=0}$ as given in (10) can be written as :

$$\mu^2_{H=0} = \frac{1}{2}\left[\sqrt{\frac{1+2\alpha}{\alpha^2\left(1+\frac{\nu^2}{p^2}\right)}} + \left\{1 + \frac{\alpha}{\alpha^2\left(1+\frac{\nu^2}{p^2}\right)}\right\}\right] \tag{39}$$

The condition of reflection of a wave of frequency $\frac{p}{2\pi}$ from the ionosphere, where the electron collisional frequency is ν, is then obtained by differentiating (39) with respect to α and equating it to zero. It is easy to find that $\mu^2_{H=0}$ would be minimum, when $\frac{p_0^2}{p^2} = 2$. When $\nu = 0$, the value of $\mu^2_{H=0}$ is zero at $\frac{p_0^2}{p^2} = 1$. This has been discussed by Ratcliffe (1959).

CONCLUSION

From an analysis of the expressions for the refractive index and absorption index, obtained for the different special cases, it is concluded that: :

(i) The effect of earth's magnetic field on the propagation of radio-waves in the ionosphere is not only to cause bi-refringence but also to change the values of the refractive index and the absorption index of the ionospheric medium.

(ii) The value of refractive index does not become zero at the point $\frac{p_0^2}{p^2} = 1$, when the collisional effects are taken into account.

(iii) The value of refractive index ($\mu_{H=0}$) is minimum at $\frac{p_0^2}{p^2} = 2$, when the collisional effects are taken into account and is zero at $\frac{p_0^2}{p^2} = 1$, when $\nu = 0$.

(iv) In the longitudinal case, both the values of refractive index and the absorption index are predominantly affected by the earth's magnetic field.

(v) In the transverse case, one set of values of $\mu_{\theta=30°}$ and $\chi_{\theta=90°}$ are the same as those of $\mu_{H=0}$ and $\chi_{H=0}$.

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APPENDIX

Whitehead (1956) obtained the following expressions for $\mu^2_{\nu<<p'}$ and $\chi_{\nu<<p'}$

$$\mu^2_{\nu<<p'} = 1 - \frac{X}{1+Q_1 y_L} \qquad \dots \quad (40)$$

and

$$\chi_{\nu<<p'} = \frac{1}{2}\, \frac{XZ(1+2Q_2\cot^2\theta)}{(1+Q_1 y_L)\sqrt{1-\dfrac{X}{1+Q_1 Y_L}}} \qquad \dots \quad (41)$$

where

$$Q_1 = \sqrt{1+a'} - \sqrt{a'}$$

$$Q_2 = \frac{a'}{\sqrt{1+a'}}\,(\sqrt{1+a'} - \sqrt{a'})$$

$$\cot\theta = \frac{p_L}{p_L}, \quad Z = \beta X$$

$$Y_L = \frac{p_L}{p_T}\,.\; X = \frac{p_0^2}{p^2}$$

Equation (40) can be rewritten as

$$\mu^2_{\nu<<p'} = 1 + \frac{1}{\alpha - \{\sqrt{1+a'} - \sqrt{a'}\}\gamma_L}$$

This is the same as given in equation (33). Equation (41) can be rewritten as

$$\chi_{\nu<<p'} = \frac{1}{2}\, \frac{\beta + \dfrac{\nu\sqrt{a'}}{p'}\left\{1 - \dfrac{\sqrt{a'}}{\sqrt{1+a'}}\right\}\gamma_L}{\left[\{\alpha - (\sqrt{1+a'} - \sqrt{a'})\gamma_L\}^2 \sqrt{1 + \dfrac{1}{\alpha - \{\sqrt{1+a'} - \sqrt{a'}\}\gamma_L}}\right.}$$

This is the same as shown in equation (34).